A Simple Problem: Solubility of $Pb(IO_3)_2$ in Distilled Water

$$Pb(IO_3)_2(s) \leftrightarrow Pb^{2+}(aq) + 2IO_3^-(aq)$$

$$K_{sp} = [Pb^{2+}][IO_3^-]^2 = 2.5 \times 10^{-13}$$

	$Pb(IO_3)_2(s)$	$\leftrightarrow$	$Pb^{2+}(aq)$	+	$2IO_3^-(aq)$
Initial Concentration	solid		0		0
Change in Concentration	solid		+x		+2x
Equilibrium Concentration	solid		x		2x

$$(x)(2x)^2 = 2.5 \times 10^{-13}$$

$$4x^3 = 2.5 \times 10^{-13}$$

$$x = 3.97 \times 10^{-5}$$

$$[Pb^{2+}] = x = 4.0 \times 10^{-5} \text{ M}$$

$$[I^-] = 2x = 7.9 \times 10^{-5} \text{ M}$$

Since one mole of $Pb(IO_3)_2$ contains one mole of Pb^{2+}, the solubility of $Pb(IO_3)_2$ is the same as the concentration of Pb^{2+}; thus, the solubility of $Pb(IO_3)_2$ is 4.0×10^{-5} M

A More Complex Problem: Solubility of $Pb(IO_3)_2$ in 0.1 M $Pb(NO_3)_2$

$$Pb(IO_3)_2(s) \leftrightarrow Pb^{2+}(aq) + 2IO_3^-(aq)$$

$$K_{sp} = [Pb^{2+}][IO_3^-]^2 = 2.5 \times 10^{-13}$$

	$Pb(IO_3)_2(s)$	$\leftrightarrow$	$Pb^{2+}(aq)$	+	$2IO_3^-(aq)$
Initial Concentration	solid		0.10		0
Change in Concentration	solid		+x		+2x
Equilibrium Concentration	solid		0.1 + x		2x

$$(0.10 + x)(2x)^2 = 2.5 \times 10^{-13}$$

$$4x^3 + 0.4x^2 = 2.5 \times 10^{-13}$$

Make Assumption!

$$[Pb^{2+}] = 0.10 + x \approx 0.10 \text{ M}$$

$$(0.10)(2x)^2 = 2.5 \times 10^{-13}$$

$$0.4x^2 = 2.5 \times 10^{-13}$$

$$x = 7.91 \times 10^{-7}$$

Check Assumption!

$$[Pb^{2+}] = 0.10 + 7.91 \times 10^{-7} \approx 0.10 \text{ M (yes!; error of 0.00079\%)}$$

$$[I^-] = 2x = 1.6 \times 10^{-6} \text{ M}$$

The solubility of $Pb(IO_3)_2$ is equal to the additional concentration of Pb^{2+} in solution, or 7.9×10^{-7} moles/liter.

An Even More Complex Problem: Solubility of $Pb(IO_3)_2$ in 1.0×10^{-4} M $Pb(NO_3)_2$

$$Pb(IO_3)_2(s) \leftrightarrow Pb^{2+}(aq) + 2IO_3^-(aq)$$

$$K_{sp} = [Pb^{2+}][IO_3^-]^2 = 2.5 \times 10^{-13}$$

	$Pb(IO_3)_2(s)$	$\leftrightarrow$	$Pb^{2+}(aq)$	+	$2IO_3^-(aq)$
Initial Concentration	solid		1.0×10^{-4}		0
Change in Concentration	solid		+x		+2x
Equilibrium Concentration	solid		$1.0 \times 10^{-4} + x$		2x

$$(1.0 \times 10^{-4} + x)(2x)^2 = 2.5 \times 10^{-13}$$

Make Assumption!

$$[Pb^{2+}] = 1.0 \times 10^{-4} + x \approx 1.0 \times 10^{-4} \text{ M}$$

$$(1 \times 10^{-4})(2x)^2 = 2.5 \times 10^{-13}$$

$$x = 2.50 \times 10^{-5}$$

Check Assumption!

$$[Pb^{2+}] = 1.0 \times 10^{-4} + 2.5 \times 10^{-5} = 1.25 \times 10^{-4} \text{ M} \neq 1.0 \times 10^{-4} \text{ M (oops!, 25\% error)}$$

Make New Assumption!

$$[Pb^{2+}] = 1.0 \times 10^{-4} + x \approx 1.2 \times 10^{-4} \text{ M}$$

$$(1.2 \times 10^{-4})(2x)^2 = 2.5 \times 10^{-13}$$

$$x = 2.28 \times 10^{-5}$$

Check New Assumption!

$$[Pb^{2+}] = 1.2 \times 10^{-4} + 2.28 \times 10^{-5} = 1.23 \times 10^{-4} \text{ M} \approx 1.2 \times 10^{-4} \text{ M (yes!, 2.5\% error)}$$

The solubility of $Pb(IO_3)_2$ is 2.3×10^{-5} moles/liter